FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS: ANALYSIS, LOGIC AND DISCRETE MATHEMATICS

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Review
on PhD dissertation of Bayan Bekbolat “Dunkl analysis and application to inverse source problems”

The PhD dissertation by Bayan Bekbolat, titled "Dunkl analysis and application to inverse source problems," is devoted to the development of Dunkl analysis and the application of Dunkl analysis to inverse source problems related to parabolic and pseudo-parabolic type equations.

The Dunkl operator is a differential difference operator used in the study of special functions, particularly in the context of representation theory and harmonic analysis. It is named after its discoverer, Charles F. Dunkl, who introduced it in the early 1980s. The Dunkl operator is closely related to symmetric groups, root systems, and other areas of algebra and analysis. This PhD dissertation consists of three chapters that investigate pseudo-differential operators generated by the Dunkl operators, and inverse source problems associated with parabolic and pseudo-parabolic type equations involving Dunkl operators and fractional derivatives on time variables, as Caputo fractional and bi-ordinal Hilfer fractional derivatives.

In Chapter 1, the preliminary results about Dunkl analysis are presented. This includes discussions on the Dunkl operator, Dunkl kernel, Dunkl transform, their properties, and some fundamental definitions from fractional calculus.

Chapter 2 covers pseudo-differential, amplitude, adjoint, and transpose operators generated by the Dunkl operator, and their continuity properties are demonstrated in classical Schwartz spaces. This chapter also includes results on the boundedness of pseudo-differential operators generated by the Dunkl operator in L^2-type spaces under specific assumptions.

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Chapter 3 focuses on three inverse source problems: inverse source problems for heat and pseudo-parabolic equations with Caputo fractional derivatives on time variables, and for the heat equation with bi-ordinal Hilfer fractional derivatives on time variables, generated by the Dunkl operators. Well-posedness results are obtained for all three problems in Sobolev-type spaces. Additionally, stability results and examples are provided.

While one of the problems in Chapter 3 has been published, the other works remain unpublished. The results are novel, so it is expected that some publications will follow. Given the significance and novelty of the obtained results, in my opinion, this PhD dissertation meets all the requirements, and its author deserves the degree of PhD in the specialty "6D060100-Mathematics."

Yours sincerely

Michael Ruzhansky

Prof. Dr. Michael Ruzhansky

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